A working person’s guide to situation theory*

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1 Introduction

This is meant to provide an informal introduction to naive situation theory which will be useful to a working linguist or cognitive scientist who wishes to apply the theory. It may also be of interest to some situation theorists. We use Extended Kamp Notation (EKN) as developed in Barwise and Cooper (forthcoming) as well as introducing the basic elements of the standard linear notation which is to be found in the literature on situation theory and situation semantics. EKN is so called because it takes its inspiration from the notation that is used by Kamp in developing Discourse Representation Theory (DRT).

2 Some preliminaries

Here we give some of the general conventions of the EKN notation.1

2.1 Conjunction

The basic notational device of EKN is the box. A box groups together one or more items of the same sort into a conjunctive item of that same sort. Thus if \( A \) and \( B \) represent situation-theoretic objects of a sort that can be conjoined, for example propositions, then

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1 This section contains revised material from Barwise and Cooper (forthcoming)
represents their conjunction. Thus if \( p \) is the proposition that Anna smiles in situation \( s \) and \( q \) is the proposition that Maria laughs in situation \( s' \) then

\[
\text{(2)} \quad \boxed{p \quad q}
\]

is the proposition that Anna smiles in \( s \) and Maria laughs in \( s' \).

To make clear that single expressions are in fact expressions of EKN it is useful to insist that every expression of EKN have an outermost box. Thus

\[
\text{(3)} \quad \boxed{A}
\]

is a notation for \( A \). If you like you can think of this a conjunction of just one object \( A \).

We take conjunction to be commutative, i.e. it does not matter what order you write things down in a conjunction. Thus, we take the following boxes to represent the same object:

\[
\text{(4)} \quad \boxed{A \quad B} \quad \boxed{B \quad A}
\]

For example, if \( p \) and \( q \) are as above then both (5a) and (5b) represent the same proposition that Anna smiles in \( s \) and that Maria laughs in \( s' \).
We also take conjunction to be associative. Thus, we take all of the following three boxes to represent the same object:

Thus the proposition that Anna smiles in \( s \) and Maria laughs in \( s' \) and Anna sees Maria in \( s'' \) is the same as the proposition that Anna smiles in \( s \) conjoined with the proposition that Maria laughs in \( s' \) and Anna sees Maria in \( s'' \) which in turn is the same as the proposition that Anna smiles in \( s \) and Maria laughs in \( s' \) conjoined with the proposition that Anna sees Maria in \( s'' \).

In EKN, whenever objects can be conjoined, they can also be disjoined. Thus, for example, the following box represents the disjunction of \( A \) and \( B \), i.e. the proposition that is true just in case either \( A \) is true or \( B \) is true.

We take the same rules of commutativity and associativity apply to disjunction as well as conjunction.
Exercise 1

1. Suppose that $p$ is the proposition that Anna likes Claire in situation $s$, $q$ is the proposition that Claire likes Anna in situation $s'$ and that $r$ is the proposition that Anna hugs Claire in situation $s'$. What propositions do the following represent?

(a) $p \quad q$

(b) $p \quad q \lor r$

(c) $p \quad q \lor r$

2. Which of the following pairs of conjoined and disjoined objects are identical?

(a) $A \quad B \lor C \lor D$

(b) $A \quad B \lor C$

4
2.2 Restrictions

We may associate restrictions with an object. In many cases such restrictions constitute a proposition. Suppose for example that \( p(a) \) is the proposition that \( a \) is bald in situation \( s \) and \( q(a) \) is the proposition that \( a \) is the one and only king of France in situation \( s' \). Then we might represent the proposition that the king of France (in \( s' \)) is bald (in \( s \)) by

\[
\begin{array}{cc}
\text{(8)} & p(a) \\
& q(a)
\end{array}
\]

If the restriction \( q(a) \) is false then (8) does not represent anything. It is undefined. If the proposition \( q(a) \) is true then (8) represents the proposition \( p(a) \) but provides in addition the information that \( q(a) \) is true.

This notion of restriction corresponds to what has often been called presupposition in the semantic literature (though we will see below that the kinds of restrictions that we allow generalize to cases that might not normally be called presupposition).

Exercise 2

1. Suppose, as is often assumed, that the proposition \( p \), that John regrets that Griselda left presupposes \( q \), that Griselda left. How might you represent this proposition in EKN?

2. Do the following two pieces of EKN notation always/sometimes/never represent the same object? If sometimes, can you tell the conditions under which they do represent the same object?

\[
\begin{array}{cc}
\text{(a)} & p \\
& q \\
& p
\end{array}
\]

\[
\begin{array}{cc}
\text{(b)} & p \\
& p \\
& p
\end{array}
\]

2.3 Abstraction

In addition to the ordinary kinds of objects you might expect we also allow our notation to talk about parametric objects. These are like ordinary objects except that they contain parameters in place of some “real” objects. We normally represent parameters with capital letters. Suppose, for example, that we
represent the proposition that Anna likes Maria and Maria seems to like Anna as \( p(a, m) \). We could then represent the parametric proposition “X likes Y and Y seems to like X” as \( p(X, Y) \). The idea is that these parameters could be *anchored* to various different individuals to obtain different propositions. An important thing that you can do with parameters is *abstract* over them. Thus, for example, if we have the proposition \( p(X, Y) \) we can abstract over its parameters to form the proposition abstract

\[
\begin{array}{c}
  r_1 \rightarrow X, r_2 \rightarrow Y \\
p(X, Y)
\end{array}
\]

This is an abstract that has two roles indexed by \( r_1 \) and \( r_2 \). If you want intuitive characterizations of the roles try “liking and seeming to be liked by” for \( r_1 \) and “being liked and seeming to like” for \( r_2 \). We might think of role indices as being words like “agent” and “patient” when appropriate. We might use role indices for the kind of indices that linguists often talk about, i.e. we could use numbers as the role indices.

One important property of abstracts is that it does not matter which parameter symbols you use to represent the roles – the particular parameters have been abstracted away from. Thus (9) represents the same object as (10)

\[
\begin{array}{c}
  r_1 \rightarrow Z, r_2 \rightarrow W \\
p(Z, W)
\end{array}
\]

In addition, the order in which the parameter symbols together with their role indices are written down does not make any difference. Thus (9) is identical with (11).

\[
\begin{array}{c}
  r_2 \rightarrow Y, r_2 \rightarrow X \\
p(X, Y)
\end{array}
\]

Intuitively abstracts might be thought of as objects with holes in them, their roles, which need to have objects assigned to their roles in order to fill the holes and make an object which is not an abstract. We will introduce assignments to the roles. An appropriate assignment for (9) is one that assigns objects to all of the role indices of the abstract (and possibly to other objects as well).
We can apply abstractions to fill in the holes with the objects assigned to the role indices by the assignment. The operation which application uses is known as substitution. We represent the application as in (13).

\[ \begin{align*}
    & r_1 \rightarrow X, r_2 \rightarrow Y \\
    & p(X, Y) \\
\end{align*} \]

(13) represents the same proposition as (14).

\[ p(a, c) \]

(14) is the result of removing the tab from the top of (9) and replacing the parameters \( X \) and \( Y \) with \( a \) and \( c \) as indicated by the assignment. This operation is known as \( \beta \)-conversion.

We can also abstract over objects associated with a restriction. Suppose, for example, that \( p(X) \) is the parametric proposition that \( X \) is asleep and that \( q(X) \) is the proposition that \( X \) is named Maria. Then (15) is the parametric proposition that \( X \) is asleep where \( X \) is named Maria.

\[ \begin{align*}
    & p(X) \\
    & q(X) \\
\end{align*} \]

Now we can form a proposition abstract from this, say:

\[ \begin{align*}
    & 16 \rightarrow X \\
    & p(X) \\
    & q(X) \\
\end{align*} \]
We might think of this as corresponding to “$X_{16}$ is asleep” where $X_{16}$ is named Maria. The restriction now places an additional condition on what the appropriate assignments for this abstract are. They have to be assignments that assign to 16 someone or something named Maria. Thus, for example, if $m$ represents my daughter Maria and $a$ represents my daughter Anna, (17a) is an appropriate assignment for (16) whereas (17b) is not.

\[(17)\]
\[
\begin{align*}
  \text{a.} & \quad [16 \to m] \\
  \text{b.} & \quad [16 \to a]
\end{align*}
\]

Since for many purposes it does not matter precisely which indices we use it is convenient to adopt the standard logical convention that of using the whole numbers beginning with 1 when we do not explicitly mention otherwise. Thus we abbreviate (18a) as (18b).

\[(18)\]
\[
\begin{align*}
  \text{a.} & \quad 1 \to X, 2 \to Y \\
  & \quad p(x, y) \\
  \text{b.} & \quad p(x, y)
\end{align*}
\]

Exercise 3

1. Which of the following abstracts are identical?

\[(a)\]
\[
\begin{align*}
  0 \to X, 1 \to Y \\
  p(x, y)
\end{align*}
\]

\[(b)\]
\[
\begin{align*}
  1 \to X, 0 \to Y \\
  p(x, y)
\end{align*}
\]
2. Suppose we have the following abstract:

<table>
<thead>
<tr>
<th>agent → X, patient → Y</th>
<th>p(X, Y)</th>
<th>q(X)</th>
<th>r(Y)</th>
</tr>
</thead>
</table>

Suppose further that \( q(X) \) is the parametric proposition that \( X \) is male and \( r(Y) \) is the parametric proposition that \( Y \) is female. Let \( a \) and \( b \) represent Alan and Barry (who are male) and \( c \) and \( d \) represent Claire and Doris (who are female). Assuming for now that \( p \) has no additional restrictions which of the following assignments are appropriate for this abstract? Why or why not? For those which are appropriate say what the result of applying this abstract to the assignment is.

(a) \[
\begin{align*}
\text{agent} & \rightarrow a \\
\text{patient} & \rightarrow c
\end{align*}
\]

(b) \[
\begin{align*}
\text{agent} & \rightarrow a \\
\text{patient} & \rightarrow b
\end{align*}
\]

(c) \[
\begin{align*}
\text{agent} & \rightarrow a \\
\text{patient} & \rightarrow c \\
45 & \rightarrow b \\
h & \rightarrow c
\end{align*}
\]

(d) \[
\begin{align*}
X & \rightarrow a \\
Y & \rightarrow c
\end{align*}
\]

3 Naive situation theory

Having introduced these preliminaries we can now embark on a description of the situation theoretic universe. The description we will give here will be informal (hence the designation “naive” situation theory)
and hopefully intuitive. At each step we will attempt to give illustrations which suggest how the various objects in situation theory might be used for semantic analysis. Thus we will give an informal introduction not only to situation theory but also to situation semantics. For a more formal characterization of situation theory in the style we are adopting here see Barwise and Cooper (forthcoming).

3.1 Infons

The first kind of objects we introduce are infons (also called states of affairs, soas, or possible facts) in the literature. The term “infon” is meant to suggest that these objects are the basic units of information. The term “possible fact” suggests the intuition that these units of information may or may not be supported in the world, i.e. they may or may not hold in some situation. (19a) is an example of an infon represented in EKN notation. (19b) is the same infon represented in standard linear notation.

(19) a. \[ r(a, b) \]

b. \[ \langle r, a; b; 1 \rangle \]

These are examples of basic infons. Each basic infon has a relation and a number of arguments. In addition it is either positive or negative. This is a positive infon. This is represented in the linear notation by including the polarity 1.

Relations are objects in the situation theoretic universe which have a type of assignments associated with them called their appropriateness conditions or restrictions.\(^2\) Relations are a particular kind of abstract and thus they have indexed roles and restrictions on their assignments as we described in Section 2.3. The type of assignments associated with a relation characterize what assignments are appropriate to it. This means that a more explicit and correct notation for infons would mention the assignments and indices explicitly as in example (20).

(20) a. \[ r(1 \rightarrow a, 2 \rightarrow b) \]

b. \[ \langle r, 1 \rightarrow a, 2 \rightarrow b; 1 \rangle \]

\(^2\)Types are a kind of situation theoretic object which we will encounter below.
However, we will only do this when absolutely necessary. However, it is important to remember that this notation only represents an infon when the assignment is appropriate to the relation. For example, suppose that $b$ represents the colour blue, $h$ represents the quality of honesty, and $r$ represents the relation admire. Under standard intuitive assumptions (21) does not represent an infon because blue and honesty are not an appropriate assignment to the roles of admire.

\[(21) \quad r(b, h)\]

Appropriateness conditions correspond in this example to what philosophers call sortal restrictions and what linguists have called selectional restrictions. An important aspect of the treatment of this phenomenon in situation theory is that the restrictions are not dependent on language. As McCawley (1970) pointed out if somebody claims that their toothbrush is pregnant we do not send them to a remedial English class for help, since we assume that they have some more fundamental misunderstanding of the nature of the world that does not have to do with the grammar of English.

In addition to positive basic infons there are also negative basic infons. Represented in EKN and the standard linear notation in (22).

\[(22) \quad \begin{align*}
(a) \quad & \neg r(a, b) \\
(b) \quad & \langle r(a, b; 0) \rangle
\end{align*}\]

Only basic infons may be negated. A negative infon corresponds to the presence of negative information in a situation (as opposed to the absence of positive information). When two basic infons are the same except that they have different polarity we say that one is the dual of the other. If $\sigma$ is a basic infon we write its dual as $\overline{\sigma}$.

Given any two infons there is another (non-basic) infon which the their conjunction and another which is their disjunction:

\[(23) \quad \begin{align*}
a. \quad & \sigma \\
\tau \quad & \end{align*}\]
b. \( \sigma \land \tau \)

(24) a. \( \sigma \lor \tau \)

b. \( \sigma \lor \tau \)

**Inference**

Under this heading in each section we will point out some rules of inference that situation theorists normally assume. Some of these are regarded as options which we can choose to have or not. We will normally make clear those which are regarded optional.

The rules of inference are in each case divided into kinds. The first are those that have to do with equality, i.e. they indicate when two expressions in our notation represent the same situation theoretic object. We use the sign ‘\( = \)’ to show equality. Thus \( p = q \) means that \( p \) and \( q \) represent the same object. The other kind have to do with equivalence. Equivalence means something different for each sort of object that we introduce and so part of characterizing inference is to say what equivalence consists in. We use the sign ‘\( \approx \)’ to represent equivalence. Thus \( p \approx q \) means that \( p \) and \( q \) are equivalent.

**Equality** Infons have the commutativity and associativity properties discussed in section 2.1.

This means, for example, that if we ignore time we will not get different infons corresponding to Grice’s (1989) famous example sentences “John took of his trousers and went to bed” and “John went to be and took off his trousers”. Examples (25a) and (25b) represent the same infon.

(25) a. take-off-trousers(\( j \))
go-to-bed(\( j \))

b. go-to-bed(\( j \))
take-off-trousers(\( j \))
If, however, as is normally assumed in situation semantics, we say that such relations have argument roles for times, then we can say that the order of utterance of the natural language sentences is reflected in an ordering on the times associated with the two infons. Thus we might have the two distinct infons represented in (26).

\begin{tabular}{|c|c|}
\hline
(26) & \begin{align*}
a. & \text{take-off-trousers}(j, t_1) \\
goto-bed(j, t_2) & t_1 < t_2 \\
b. & \text{take-off-trousers}(j, t_2) \\
goto-bed(j, t_1) & t_1 < t_2 \\
\end{align*} \\
\hline
\end{tabular}

\textbf{Equivalence} Two infons are equivalent implies that they hold in exactly the same situations. This is expressed formally as

$$\sigma \equiv \tau \text{ implies } \forall s[\models \sigma \iff \models \tau]$$

In order to understand what this means we must first learn about situations and the relationship between situations and infons. This we do in the next section.

\textbf{Exercise 4}

1. Taking your normal intuitions about the words used in the following examples explain which of the following represent infons. For those which you do not think represent infons, explain why not. (We will continue to leave out time roles for ease of exposition so do not count the absence of a time as a reason for something not being an infon.)

\begin{tabular}{|c|}
\hline
(a) & \begin{align*}
& \text{love(Anna,Maria)} \\
& \neg \text{love(Maria,Anna)} \\
\end{align*} \\
\hline
(b) & \begin{align*}
& \text{married(Jon,Mary-Ellen)} \\
& \neg \text{married(Mary-Ellen,Jon)} \\
\end{align*} \\
\hline
\end{tabular}
3.2 Situations

While we allow situations to be denoted by letters in our notation, we do not have any special kind of boxes to represent situations. There is an important relation that holds between situations and infons. We talk of a situation supporting an infon or equivalently of an infon holding in a situation. We can also think of an infon as representing a type of situation. Thus it is equivalent to say that a situation $s$ supports an infon $\sigma$ or that $\sigma$ holds in $s$ or that $s$ is of the type $\sigma$. (Not every situation theorist will agree that the last form of words should be used to mean exactly the same as the other two.) We will normally use the locution “$s$ supports $\sigma$”. In symbols this is standardly written $s \models \sigma$. It is standardly assumed that situations are identified by the infons they support, i.e. you will not find two distinct situations that support exactly the same infons, though again this is something that some situation theorists assume and others do not.

The leading intuition behind the notion of situation is that it is a part of the world which a cognitive agent might perceive. Different agents can perceive different kinds of situations. Consider the difference between a human being and a frog (as discussed in Barwise and Perry, 1983). Frogs can only perceive situations in which there is motion, whereas human beings can also perceive static situations. One of the important insights about perception enshrined in the work on situation theory is that the parts of the world agents perceive are not defined in the way that one might initially expect. Situations are, for example, not individuated by space-time locations. The parts of the world we perceive are not for example parts of the world which support all the facts in our visual field at a particular time. As cognitive agents we are forced to concentrate our attention on parts of the world which support particular relevant facts which we have in focus at a particular time. Thus the division of the world into situations is one that is based on information rather than space-time.

The intuition that situations might be objects perceived by cognitive agents can often lead us to make intuitive decisions about the nature of situation theory. The claim is not that every situation should be perceivable, but rather that we should include enough situations in our situation theoretic universe to account for all the cases where we would want to say that an agent perceives a situation. Let us now look at some normally assumed axioms about situations and see how they might interact with these intuitions.

**Extensionality:** $\forall s, s', \sigma [s \models \sigma \iff s' \models \sigma \implies s = s']$

**Conjunction:** $\forall s, \sigma, \sigma'[s \models \sigma \land \sigma' \iff s \models \sigma \land s \models \sigma']$
**Disjunction:** $\forall s, \sigma, \sigma'[s |= \sigma \lor \sigma'$ iff $s |= \sigma$ or $s |= \sigma'$

**Consistency:** $\forall s, \sigma[s |= \sigma$ is a basic infon and $s |= \sigma$ implies $s \not|= \overline{\sigma}$

Notice that in the conjunction axiom we have a biconditional meaning that the inference goes both ways. If $s$ supports $\sigma \land \tau$ then $s$ supports $\sigma$ and $s$ supports $\tau$ and also if $s$ supports $\sigma$ and $s$ supports $\tau$ then $s$ support $\sigma \land \tau$. This corresponds with our intuitions. If somebody sees Mary run and John run then presumably they see Mary run and they see John run. Conversely, if they see Mary run and they see John run then it seems reasonably to suppose that they see Mary run and John run. It seems that you can’t perceive the conjunction of facts without perceiving the individual facts and you can’t perceive the individual facts without perceiving the conjunction.3

With disjunction the case is a bit different however. It seems to be the case that if someone perceives a disjunctive fact, e.g. they see a situation where either John runs or Mary runs, that they have perceived a situation which supports one or the other of the facts. But should it be the case that if they perceive one or other of the facts that they have actually perceived a situation which supports the disjunction? The axiom we have stated requires this too. Under normal assumptions this would mean that all situations would support infinitely many infons. Suppose that Harry saw Mary run, i.e. the situation that he saw supported

\[
\text{run(Mary)}
\]

Then, if the disjunction axiom were biconditional, we could argue for any infon $\sigma$ that the situation he saw supported

\[
\text{run(Mary)} \lor \sigma
\]

and it would standardly be the case that there are infinitely many such $\sigma$. The disjunction axiom as stated is standardly assumed in situation theory, although I have some reservations about it.

Notice that we do NOT have the following axiom.

**Excluded middle** $\forall s, \sigma[s |= \sigma$ or $s |= \overline{\sigma}$

---

3 Actually one could try to argue that one could see the individual facts without somehow putting them together and I have attempted to argue this in the past. But I think it is probably incorrect.
If we were to include this axiom then situations would no longer provide enough objects to serve as objects of perception. It is possible, for example, to see Mary smoke without seeing either Charles run or not run. We perceive only part of the world whereas the axiom of the excluded middle would require that we see the whole world.

In addition to the general axioms that we have given it appears there are constraints relating to particular relations. Barwise and Perry (1983) discuss the example of kissing and touching. If \( a \) kisses \( b \) then it is the case that \( a \) touches \( b \). There is a question though of whether both infons have to be supported by the same situation. Compare this with the case of running and moving legs. It seems to be the case that if \( a \) runs then \( a \)'s legs move. However, it seems clear in this case that there are situations in which \( a \) runs but which do not contain information about their legs. For example, I can see somebody run past my house although their legs are hidden behind the garden wall. Thus the situation I perceived did not support the fact that their legs moved. Nevertheless I can come to the conclusion that their legs were moving even though this fact was not part of the situation I perceived. In this case we say that the information that their legs were moving is carried by the situation I perceived, but not supported by it.

Some versions of situation theory regard all situations as being actual. That is, the theory is only concerned with actually existing situations. Another option is to allow all possible situations into the domain of the theory. This version of the theory is closer to the theory of possible worlds which was developed in connection with modal logic and used in Montague semantics and much work that has developed from this. Some situations are regarded as worlds in situation theory. They are the ones that have the property of completeness defined below.

**Completeness** A situation \( s \) is complete iff for all relations \( r \) and appropriate assignments \( a_1, \ldots, a_n \) for \( r, s \) supports

\[
r(a_1, \ldots, a_n)
\]

or

\[
\neg r(a_1, \ldots, a_n)
\]

This means that a world has total information whereas situations in general only have partial information. This notion of world together with the assumption that there are all possible situations and not just actual ones makes it directly possible to embed a theory of possible worlds in a situation theory.\(^4\)

\(^4\) Actually, it seems that the same can be done without assuming that there are non-actual situations, but this would involve letting complete and coherent situation types correspond to possible worlds and is therefore somewhat less direct.
Exercise 5

1. How can we show that a situation that supports the following infon does not exist according to the axioms above?

\[
\begin{array}{c}
\sigma \\
\sigma \vee \tau \\
\tau
\end{array}
\]

Does that mean that this is not an infon?

2. Do you think that if a situation supports

\[
\text{ kissing } (a, b)
\]

then that same situation must support

\[
\text{ touching } (a, b)
\]

Explain why or why not. If not, can you think of any similar cases where you would want to say that the same situation must support both infons? Or can you think of general arguments that this will never be the case? Can you think of any characteristics of infons which would suggest that they would never be required to be supported by the same situation?

3.3 Propositions

There are two kinds of basic propositions which we introduce as objects into the situation theoretic universe. The first concerns situations and infons and can be read as “the proposition that s supports \( \sigma \)”.

In (27) we give the EKN notation followed by the standard linear notation as usual.

\[
(27) \quad \begin{array}{c}
\text{ a.}
\end{array}
\]

\[
\begin{array}{c}
s \\
\sigma
\end{array}
\]
b. \((s \models \sigma)\)

Such a proposition is true just in case \(s\) supports \(\sigma\).

One way we can think of interpreting natural language sentences is in a similar fashion as is done, say, in Montague semantics, except that where Montague used possible worlds we could use situations.\(^5\) Thus we could interpret sentences relative to situations just as Montague interprets sentences relative to possible worlds. Thus if we interpreted the sentence “Anna hugs Maria” relative to situation \(s\) we might say that its content is

\[(28)\]

\[
\begin{array}{c}
  s \\
  \text{hug}(\text{Anna}, \text{Maria})
\end{array}
\]

The second kind of basic proposition involves some object and a type.

\[(29)\]

\[
\begin{array}{c}
  b \\
  T
\end{array}
\]

a. \((b : T)\)

b. \((s \models \sigma)\)

This is true just in case the object is of the type. Types are a kind of abstract about which we will have more to say later on.

**Proposition conjunction, disjunction and negation**

For any two propositions \(p\) and \(q\) we have their conjunction and disjunction represented below in EKN and standard linear notation.

\(^5\) This can be viewed as a generalization of Montague’s treatment since possible worlds can be viewed as a subclass of the class of situations, given the assumptions discussed in section 3.2.
(30) a. \[ \begin{array}{c}
  p \\
  q 
\end{array} \]

b. \[ p \land q \]

This is true just in case \( p \) is true and \( q \) is true.

(31) a. \[ \begin{array}{c}
  p \lor q 
\end{array} \]

b. \[ p \lor q \]

This is true just in case at least one of \( p \) and \( q \) is true.

We also have negation of basic and non-basic propositions.

(32) a. \[ \neg \begin{array}{c}
  s \\
  \sigma 
\end{array} \]

b. \( (s \not\models \sigma) \)

c. \[ \neg \begin{array}{c}
  b \\
  T 
\end{array} \]

d. \( (b \not\models T) \)
The negation of a proposition $p$ is true just in case $p$ is not true.

**Truth**

Here we gather together the truth conditions for propositions which we stated informally above.

1. $\sigma$ is true iff $s \models \sigma$

2. $T$ is true iff $b$ is of type $T$

3. $p \land q$ is true iff $p$ is true and $q$ is true

4. $\neg p \lor q$ is true iff $p$ is true or $q$ is true
5. \[ \neg p \] is true iff \( p \) is not true

Readers familiar with propositional logic will recognize that the truth conditions for non-basic propositions in situation theory are the same as the classical truth conditions for the propositional logic connectives.

**Inference**

**Identity** We have commutativity and associativity of conjunction and disjunction as described in section 2.1.

**Equivalence** We cannot define equivalence for propositions in terms of truth. We would not want to say, for example, that all true propositions are equivalent. Nevertheless we can say something about the relationship between truth and equivalence for propositions, namely that any two equivalent propositions will have the same truth value. That is,

\[ p \approx q \text{ implies } p \text{ is true iff } q \text{ is true} \]

There follow two kinds of equivalences that we would want to have for propositions. There are others which we will not mention here.

**Identity** Any object is equivalent to itself. Thus in particular we have for propositions \( p \)

\[ p \approx p \]

**de Morgan equivalences** These correspond to de Morgan’s Laws in propositional logic.

1. \[ \neg \begin{array}{c}
   p \\
   q
\end{array} \approx \begin{array}{c}
   \neg p' \lor \neg q'
\end{array} \], where \( p \approx p' \) and \( q \approx q' \)
2. \( \neg p \approx \neg (p' \lor q') \), where \( p \approx p' \) and \( q \approx q' \)

**Equivalences bridging infonc and propositional logic**

1. \( \sigma \approx \tau \)

2. \( \sigma \lor \tau \approx \sigma \lor \tau \)

These equivalences correspond to the axioms for conjunction and disjunction given in section 3.2.

**Exercise 6**

1. Suppose we are interpreting English sentences relative to a situation \( s \) as suggested above. What two distinct non-equivalent propositions might correspond to the sentence “Anna smiled and hugged Maria or kissed her”? (Ignore the tense, unless you are feeling adventurous.) For each of the two propositions give another distinct proposition that is equivalent to it and another expression of EKN that represents the same proposition as the first one.

2. We have now introduced two kinds of negation. Basic infon negation and propositional negation. Consider the sentence “John didn’t stop at the traffic light”. Assuming there is a relation stop-at-the-traffic-light indicate what two propositions might correspond to this sentence using the two negations (one for each negation). Are they both possible interpretations of the sentence or is only
one of them correct? You may wish to consider the sentence “Mary saw John not stop at the traffic light” (due to Elisabet Engdahl) or you may find it irrelevant. Argue one way or the other.

3.4 Relations (Properties)

We pointed out in section 2.3 that relations are a particular kind of abstract. They are include in fact all those abstracts which are infon abstracts. In order to make it clear that an infon abstract is also a relation we will use a special notation when we are thinking of it as a relation. Thus (33a) and (33b) represent the same object, but we will generally use (33b) when we are thinking of it as a relation.

\[
\begin{array}{c}
\text{see}(X, Y) \\
\text{see}(Y, X)
\end{array}
\]

Note that this means that (34) is an infon.

\[
\begin{array}{c}
\text{see}(X, Y) \\
\text{see}(Y, X) \\
( r_1 \rightarrow a, r_2 \rightarrow b)
\end{array}
\]

Example (34) is an infon which is equivalent to (35).

\[
\begin{array}{c}
\text{see}(a, b) \\
\text{see}(b, a)
\end{array}
\]

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That is, a situation will support (34) if and only if it supports (35). One use of maintaining the distinction between (34) and (35) while maintaining their equivalence is seen in the application to situation theoretic grammar where we not only formalize semantics in terms of situation theory but also other parts of the grammar. In that case we would want to say that utterances of words and phrases are used as role indices on the relation. Thus (34) might correspond to Anna saw and was seen by Maria. In this case \( r_1 \) would be an utterance of the word ‘Anna’ and \( r_2 \) would be an utterance of the word ‘Maria’. (35) on the other hand not only has a structure which is quite different from the natural language sentence but also has a relation whose role indices are different (by convention 1 and 2 since they are not mentioned).

Inference

**Equality**  We have the same equalities as we had for abstracts, as relations are abstracts. That is,

\[
\begin{align*}
& r_1 \rightarrow X, r_2 \rightarrow Y \quad = \quad r(X, Y) \\
& r_1 \rightarrow W, r_2 \rightarrow Z \quad = \quad r(W, Z) \\
& r_2 \rightarrow Y, r_1 \rightarrow X \quad = \quad r(X, Y)
\end{align*}
\]

Equivalence

**Equivalence of support**  \( r \approx r' \) implies \( r \) and \( r' \) have the same appropriateness conditions on assignments and

\[
\forall s, a_1, \ldots, a_n \quad s \models r(a_1, \ldots, a_n) \iff s \models r'(a_1, \ldots, a_n) \quad \text{and} \quad s \models \neg r(a_1, \ldots, a_n) \iff s \models \neg r'(a_1, \ldots, a_n)
\]

\( \beta \)-equivalence

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Exercise 7

1. What relation might the following English phrases correspond to?
   (a) runs and jumps
   (b) be seen by
   (c) likes or admires William
   (d) John hates and Barry detests (pizza)

2. Are the following two supported by exactly the same situations?

Why or why not?

3.5 Types

Types also are abstracts. They are proposition abstracts. We use a special notation for them in the same way that we do for relations. Thus (36a) and (36b) are different representations of the same object.
Now we have (37) as a proposition.

This proposition is equivalent but not identical to (38).

This means that (37) is true just in case (38) is true.

Keeping these two propositions distinct, though equivalent, can be useful when analyzing propositional attitudes such as belief. For example, Frege pointed out that it is possible for a rational person to believe
that Hesperus is not identical with Phosphorus while one would not expect a rational person to believe
that Hesperus is not identical with Hesperus. However, the difference is puzzling since both Hesperus
and Phosphorus refer to the planet Venus. If both sentences picked out the same proposition, e.g. (39), it
would be difficult to explain the difference.

(39) \[ \neg \text{identical} \]

If on the other hand we say that the two sentences pick out the distinct propositions (40a) and (40b)
which are nevertheless equivalent to (39) then we can begin to see an intuitive treatment of what is going
on.

(40) a. \[ \text{‘Hesperus’} \rightarrow v, \text{‘Phosphorus’} \rightarrow v \]
\[ \text{‘Hesperus’} \rightarrow X, \text{‘Phosphorus’} \rightarrow Y \]
\[ X, Y \]
\[ \text{identical} \]

b. \[ \text{‘Hesperus’} \rightarrow v, \text{‘Hesperus’} \rightarrow v \]
\[ \text{‘Hesperus’} \rightarrow X, \text{‘Hesperus’} \rightarrow Y \]
\[ X, Y \]
\[ \text{identical} \]

**Inference**

**Equality** We have the same equalities as we had for abstracts, as types are abstracts. That is,
\[
\begin{array}{ccc}
\begin{array}{c}
X, Y \\
t
\end{array} & = & \begin{array}{c}
W, Z \\
t
\end{array} & = & \begin{array}{c}
X, Y \\
t
\end{array}
\end{array}
\]

**Equivalence** If \( t, t' \) are of arity \( n \), \( t \approx t' \) implies \( \forall a_1, \ldots, a_n \) \( \text{true} \) iff \( a_1, \ldots, a_n \)

**β-equivalence**

**Exercise 8**

1. Suppose that we now treat the content of English sentences as situation types, instead of interpreting relative to a situation. So the content of Anna smiles would be

\[
\begin{array}{c}
S \\
\text{smile(Anna)}
\end{array}
\]

Do you see any advantages or disadvantages for this compared with the earlier treatment of interpreting relative to situations? Might it help or hinder when it comes to attitude sentences such as “Robin believes Anna smiles”? 

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References

Barwise, Jon and Robin Cooper (forthcoming) Simple Situation Theory and its Graphical Notation

Further reading

This is a short list of key references which will help you get oriented in situation theory and situation semantics.


Cooper, Robin (1991) Three Lectures on Situation Theoretic Grammar in *Natural Language Processing, EAIA 90, Proceedings, Lecture Notes in Artificial Intelligence, no 476*, ed. by Miguel Filgueiras, Luis Damas, Nelma Moreira, Ana Paula Tomás, Springer Verlag

